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Optimized Production Scheduling for a Continuous Flow Food Manufacturing Environment with Sequential Operations and Parallel Machines

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Agenda

- Problem
- Introduction
- Proposed Model
- Case Study
- Future Research

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The Problem

→Closing the Food Waste Gap

40%

OF TOTAL FOOD SUPPLY IN U.S. BECOMES WASTE [1]

19%

OF GLOBAL FOOD PRODUCTION IS WASTED IN HOUSEHOLDS [5]

50%

UN SDG 12.3 CALLS FOR CUTTING IN HALF PER CAPITA GLOBAL FOOD WASTE [5]

13%

OF FOOD PRODUCED IS LOST BETWEEN HARVEST AND RETAIL [5]

38%

OF FOOD WASTE COMES FROM MANUFACTURING [2]

53%

AVERAGE YIELD FOR FOOD MANUFACTURERS [3,4]

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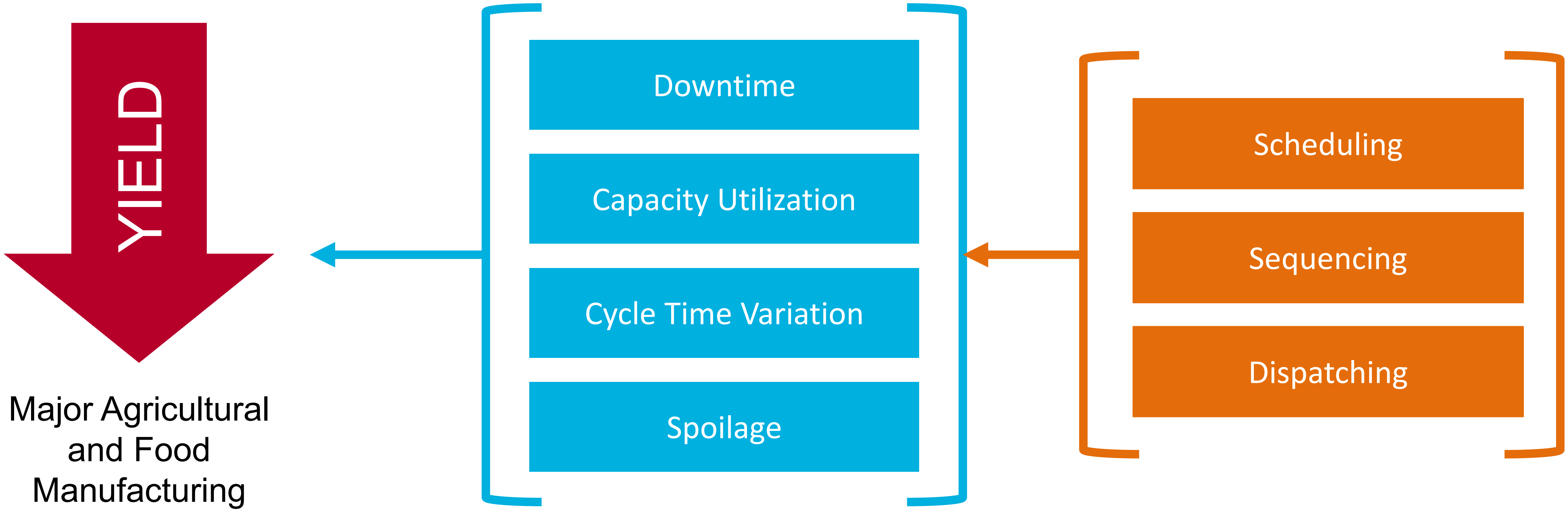
Photo: Elena/Adobe Stock / un.org

The Problem

→ Industry Partner



The Problem

→ Gaps in the Literature

Topic	Gaps in the Literature
Incorporate Demand Forecasting	Incorporates demand forecasting, production scheduling, and service level optimization.
Effects of Perishability	Addressed at supply chain network level, but not via scheduling or manufacturing at the facility level.
Multiple Conflicting Objectives	Combination of multiple objectives in scheduling optimization for food manufacturing.
Minimize Waste	Minimize food waste within the context of food manufacturing.
Dependencies Across all Manufacturing Stages	Incorporating dependencies across all stages of manufacturing processes.

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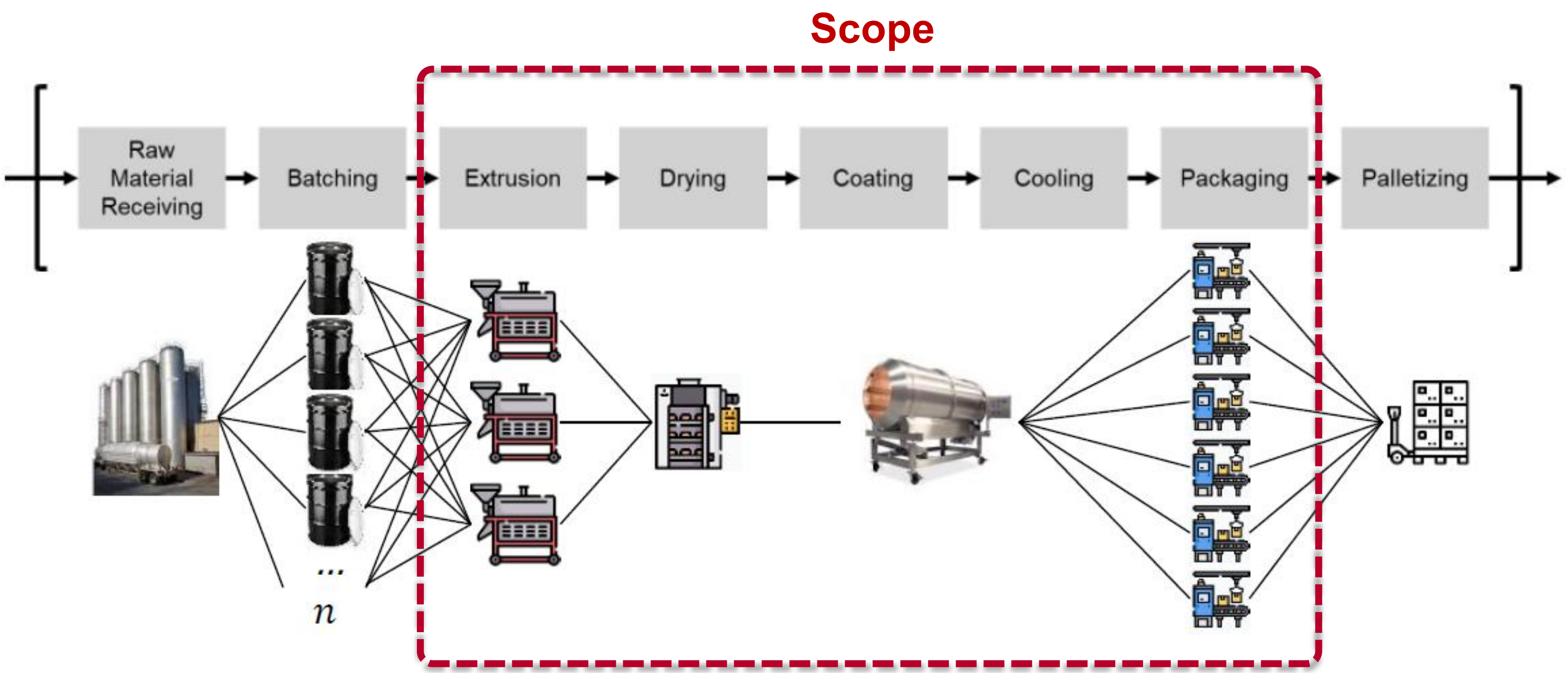
Introduction

→ Industry Client’s Process



Equipment

- 3 extruders
- 1 dryer
- 1 coater/cooler
- 6 packaging lines

Products

- 1200 SKUs
- 172 formulations
- 280 ingredients

Perishable Constraint

- Drying ≤ 16 hours from slurry formation

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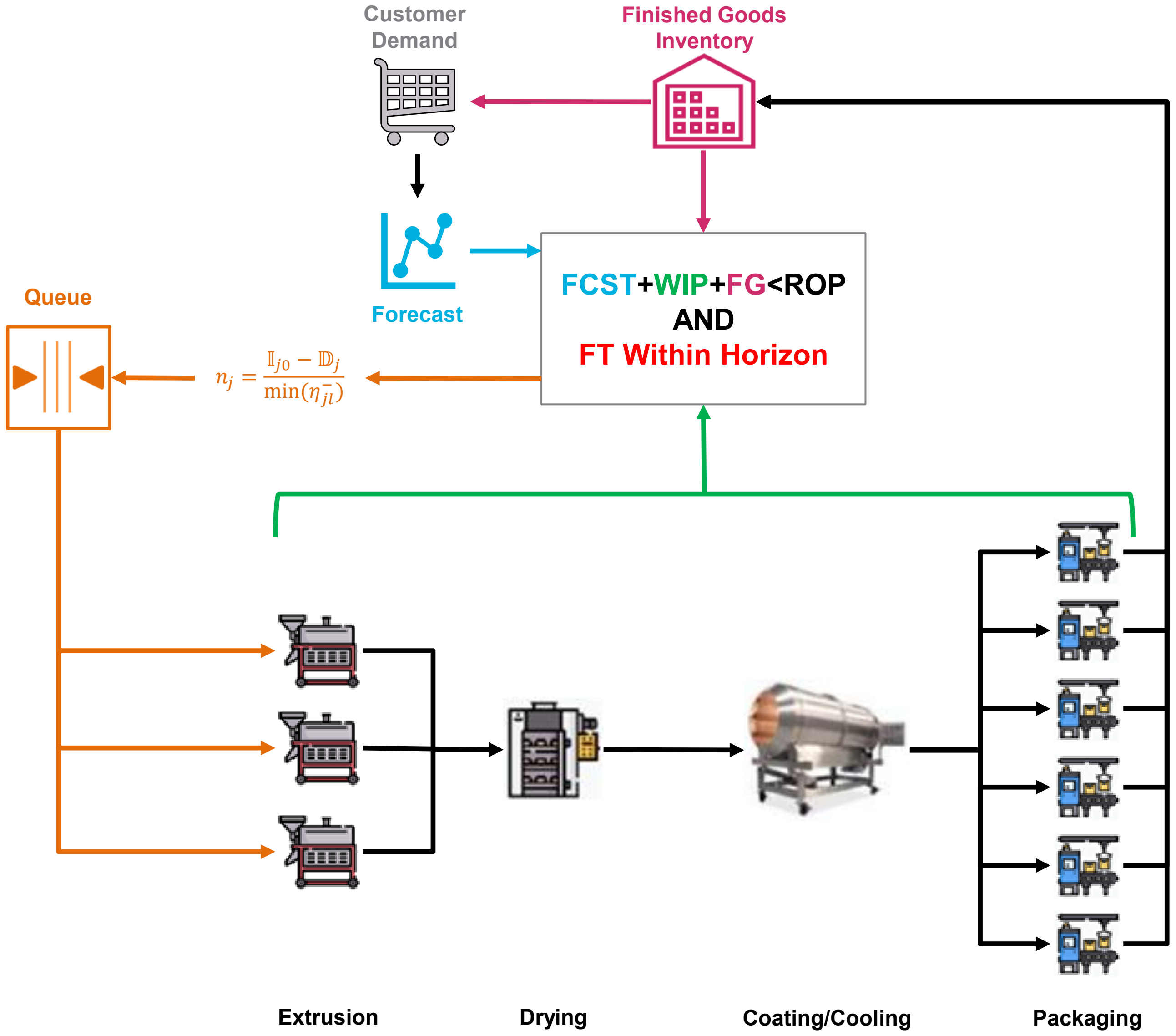
Proposed Model
→ Overview

- Preliminary simplified model
- Optimal weekly production schedule

Solution Methods

- Mixed Integer Linear Program (MILP)
- Algorithm: Branch and cut
- Programming Language: Python
  - Libraries: gurobyipy, GRB, pandas, time, numpy
- Solver: Gurobi



Proposed Model

→ Primary Assumptions

Economic Production Quantity (EPQ) batch size

$$\eta_{j\bar{l}} = \frac{1}{\xi \nu \varphi \chi} \times \sqrt{\frac{2\mathbb{D}_j \mathbb{c}^{Setup}}{h_j\left(1 - \frac{\bar{\mathbb{d}}_j}{\alpha_{jl}}\right)}}$$

Notation	Description
$\eta_{j\bar{l}}$	Input of finished good j on extruder l
ξ	Production yield of extrusion
ν	Production yield of drying machine
φ	Production yield of coating machine
χ	Production yield of packaging machine
$\mathbb{D}_j$	Annualized demand of finished good j
$\mathbb{c}^{Setup}$	Generalized setup cost ($29.52 per hour)
h_j	Inventory holding cost of finished good j
$\bar{\mathbb{d}}_j$	Average hourly demand of finished good j
α_{jl}	Production rate of finished good j on extruder l

Proposed Model

→ Parameters, Decision Variables, Constraints

Parameters	Decision Variables	Constraints
Availability	Start Time	Task Assignment
Demand	Start and Finish Times	Sequencing
Inventory	Machine Assignment	Precedence
Perishability	Inventory	Runtime
Costs		Job Initiation
Production		Inventory
		Perishability

Proposed Model
→ Primary Assumptions

Choice of Reorder Point (ROP) Method

- Static (level set by user)
- Dynamic

$$R_j = \mathbb{d}_j + Z\sigma_j^{\bar{\mathcal{L}}_j}$$

$$\mathbb{d}_j = \bar{\mathbb{d}}_j \bar{\mathcal{L}}_j$$

$$\sigma_j^{\bar{\mathcal{L}}_j} = \sqrt{\sigma_j^2 \bar{\mathcal{L}}_j}$$

$$\bar{\mathcal{L}}_j = \frac{1}{\max(K^j, 1)} \sum_k^{K^j} (\rho_k^+ - \epsilon_k)$$

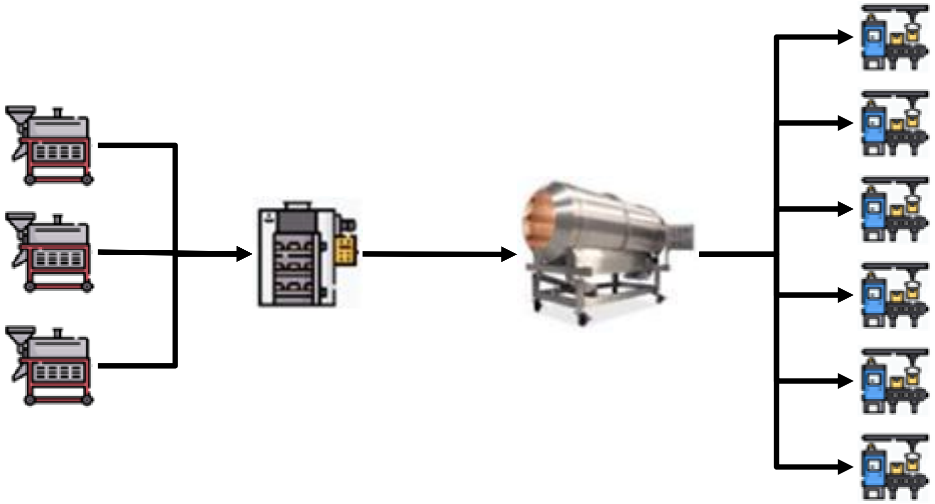
Notation	Description
R_j	Reorder point of finished good j
$\mathbb{d}_j$	Demand of finished good j during replenishment lead time
Z	Z-score (based on service level, ψ_j)
$\sigma_j^{\bar{\mathcal{L}}_j}$	Standard deviation of demand of finished good j during replenishment lead time
$\bar{\mathcal{L}}_j$	Average lead time of finished good j
σ_j^2	Variance of demand of finished good j
K^j	Set of jobs (K) producing finished good j
ρ_k^+	Finish time of job k
ϵ_k	Start time of job k

Proposed Model

→ Primary Assumptions

One job = One finished good

- Finished good (j) = Package type + Recipe (i)
- One job assigned to at most one machine
- Real case
  - Extrude, dry, coat as recipe
  - Only at packaging does it become a finished good
  - Stored as a finished good



Proposed Model

→ Objective Function

$$Min\ \mathbb{C} = \sum_{l}\sum_{k}\sum_{t}\left(\mathbb{c}^{Setup}u_{klt} + \mathbb{c}^{Labor}u_{klt} + \mathbb{c}_{l}^{Ext}u_{klt}\right) + \sum_{d}\sum_{k}\sum_{t}\left(\mathbb{c}^{Setup}o_{kldt} + \mathbb{c}^{Labor}o_{kldt} + \mathbb{c}_{d}^{Dry}o_{kldt}\right)$$

$$+ \sum_{c}\sum_{k}\sum_{t}\left(\mathbb{c}^{Setup}g_{klct} + \mathbb{c}^{Labor}g_{klct} + \mathbb{c}_{c}^{Coat}g_{klct}\right) + \sum_{m}\sum_{k}\sum_{t}\left(\mathbb{c}^{Setup}p_{klmt} + \mathbb{c}^{Labor}p_{klmt} + \mathbb{c}_{m}^{Pack}p_{klmt}\right)$$

Manufacturing Cost

$$+ \sum_{j}\sum_{t} h_j \mathbb{I}_{jt} + \sum_{j}\sum_{t} h_j^S \mathbb{I}_{jt}^S$$

Inventory & Stockout Cost

$$+ \sum_{j}\sum_{t}\left(\mathbb{c}_j^{Order} y_{jt} + \mathbb{c}_j^{Safety} s_j\right)$$

Service Level Cost

$$+ \sum_{t}\sum_{l}\sum_{k}\sum_{i} \mathbb{c}_{Waste}^{Ext}\, \eta_{kl}^{-}\xi_{il}u_{klt} + \sum_{t}\sum_{d}\sum_{k}\sum_{i} \mathbb{c}_{Waste}^{Dry}\, \varepsilon_{kd}^{-}\nu_{id}o_{kldt} + \sum_{t}\sum_{c}\sum_{k}\sum_{i} \mathbb{c}_{Waste}^{Coat}\, \zeta_{kc}^{-}\varphi_{ic}g_{klct} + \sum_{t}\sum_{m}\sum_{k}\sum_{j} \mathbb{c}_{Waste}^{Pack}\, \vartheta_{km}^{-}\chi_{jm}p_{klmt}$$

Waste Cost

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Case Study

→ Scenarios

- Simulated data representative of industry partner data set
- Illustrative example

Scenario	Number of Finished Goods	Service Level	Demand Horizon	Finished Good (j)	Starting Inventory (lbs)	Forecast (lbs)	Work Schedule	
							Hours	Days
1	1	99%	1 Week	3	-	2,000	8	5
2	1	99%	1 Week	3	-	10,000	8	5
3	1	99%	1 Week	3	500	10,000	8	5
4	1	99%	1 Week	3	1,000	10,000	8	5
5	1	99%	1 Week	3	2,500	10,000	8	5
6	1	99%	1 Week	3	500	2,000	8	5
7	1	99%	1 Week	3	1,000	2,000	8	5
8	1	99%	1 Week	3	2,500	2,000	8	5

Case Study

→ Results

Scenario	Starting Inventory (lbs)	Forecast (lbs)	Running Time (min)	Optimal Solution	Cost per Production Job	MIP Gap	Production Jobs	Amount Produced (lbs)	Reorder Point (lbs)	Average Lead Time (hrs)
1	-	2,000	4.19	$ 1,816.32	$ 1,816.32	0.00%	1	2,434.58	450.00	9.00
2	-	10,000	8.19	$ 4,891.98	$ 2,445.99	4.33%	2	10,966.66	3,125.00	12.50
3	500	10,000	16.24	$ 4,682.65	$ 2,341.33	3.58%	2	10,966.66	3,250.00	13.00
4	1,000	10,000	20.95	$ 4,593.32	$ 2,296.66	2.93%	2	10,966.66	3,125.00	12.55
5	2,500	10,000	15.62	$ 3,635.33	$ 3,635.33	1.71%	1	937.31	2,750.00	11.00
6	500	2,000	3.81	$ 1,729.41	$ 1,729.41	0.00%	1	2,434.58	950.00	19.00
7	1,000	2,000	6.74	$ 1,240.00	-	3.88%	0	-	-	-
8	2,500	2,000	0.02	$ 1,180.00	-	0.00%	0	-	-	-

Case Study

→ Illustrative Examples

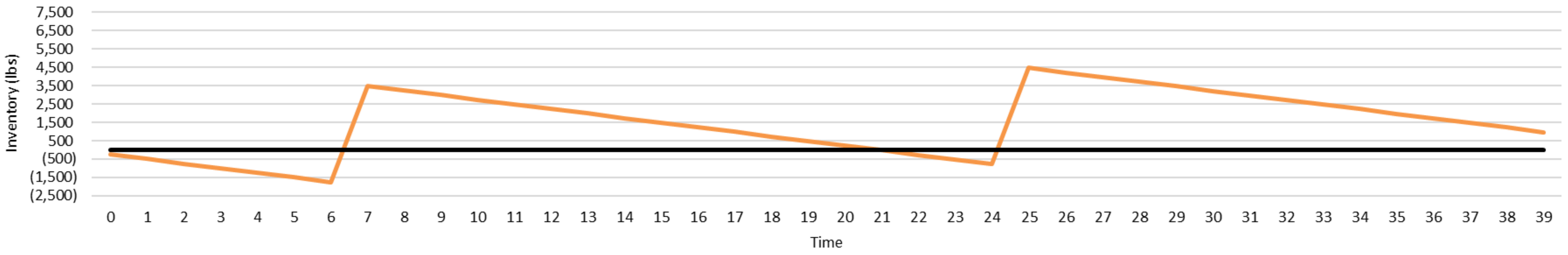
Scenario	Starting Inventory (lbs)	Forecast (lbs)	Running Time (min)	Optimal Solution	Cost per Production Job	MIP Gap	Production Jobs	Amount Produced (lbs)	Reorder Point (lbs)	Average Lead Time (hrs)
1	-	2,000	4.19	$ 1,816.32	$ 1,816.32	0.00%	1	2,434.58	450.00	9.00
2	-	10,000	8.19	$ 4,891.98	$ 2,445.99	4.33%	2	10,966.66	3,125.00	12.50
3	500	10,000	16.24	$ 4,682.65	$ 2,341.33	3.58%	2	10,966.66	3,250.00	13.00
4	1,000	10,000	20.95	$ 4,593.32	$ 2,296.66	2.93%	2	10,966.66	3,125.00	12.55
5	2,500	10,000	15.62	$ 3,635.33	$ 3,635.33	1.71%	1	937.31	2,750.00	11.00
6	500	2,000	3.81	$ 1,729.41	$ 1,729.41	0.00%	1	2,434.58	950.00	19.00
7	1,000	2,000	6.74	$ 1,240.00	-	3.88%	0	-	-	-
8	2,500	2,000	0.02	$ 1,180.00	-	0.00%	0	-	-	-

Scenario 2

Starting Inventory = 0 lbs | Forecast = 10,000 lbs

		Week	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		Day	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3
		Hour	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4
Scenario	Job	T	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
2	0	Extrude	1	1	1	1																
		Dry					0															
		Coat						0														
		Package							1	1												
2	1	Extrude																			1	1
		Dry																				
		Coat																				
		Package																				

		Week	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		Day	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5
		Hour	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
Scenario	Job	T	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
2	0	Extrude																				
		Dry																				
		Coat																				
		Package																				
2	1	Extrude	1	1																		
		Dry			0																	
		Coat				0																
		Package					1	1														

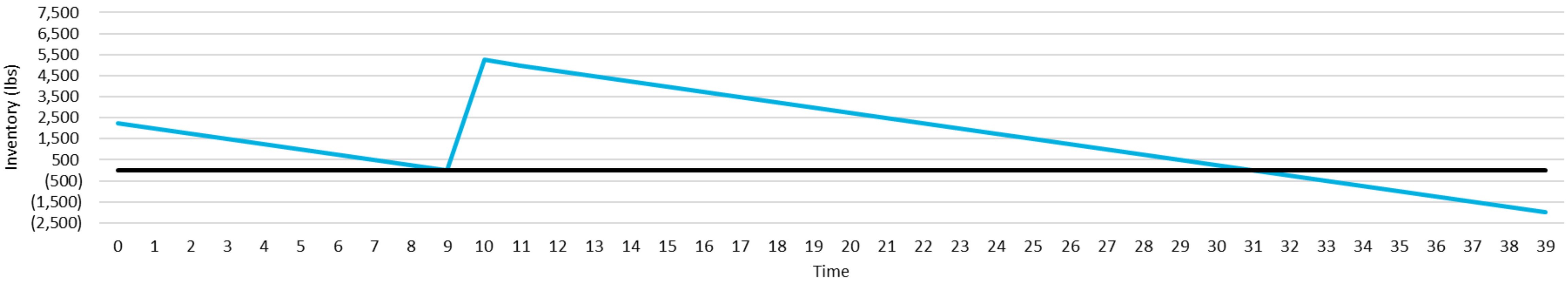


Case Study
→ Illustrative Examples

Scenario	Starting Inventory (lbs)	Forecast (lbs)	Running Time (min)	Optimal Solution	Cost per Production Job	MIP Gap	Production Jobs	Amount Produced (lbs)	Reorder Point (lbs)	Average Lead Time (hrs)
1	-	2,000	4.19	$ 1,816.32	$ 1,816.32	0.00%	1	2,434.58	450.00	9.00
2	-	10,000	8.19	$ 4,891.98	$ 2,445.99	4.33%	2	10,966.66	3,125.00	12.50
3	500	10,000	16.24	$ 4,682.65	$ 2,341.33	3.58%	2	10,966.66	3,250.00	13.00
4	1,000	10,000	20.95	$ 4,593.32	$ 2,296.66	2.93%	2	10,966.66	3,125.00	12.55
5	2,500	10,000	15.62	$ 3,635.33	$ 3,635.33	1.71%	1	937.31	2,750.00	11.00
6	500	2,000	3.81	$ 1,729.41	$ 1,729.41	0.00%	1	2,434.58	950.00	19.00
7	1,000	2,000	6.74	$ 1,240.00	-	3.88%	0	-	-	-
8	2,500	2,000	0.02	$ 1,180.00	-	0.00%	0	-	-	-

Scenario 5
Starting Inventory = 2,500 lbs | Forecast = 10,000 lbs

		Week	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		Day	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5
		Hour	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
Scenario	Job	T	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
5	0	Extrude		0	0	0	0																																			
		Dry								0																																
		Coat									0																															
		Package										1	1																													



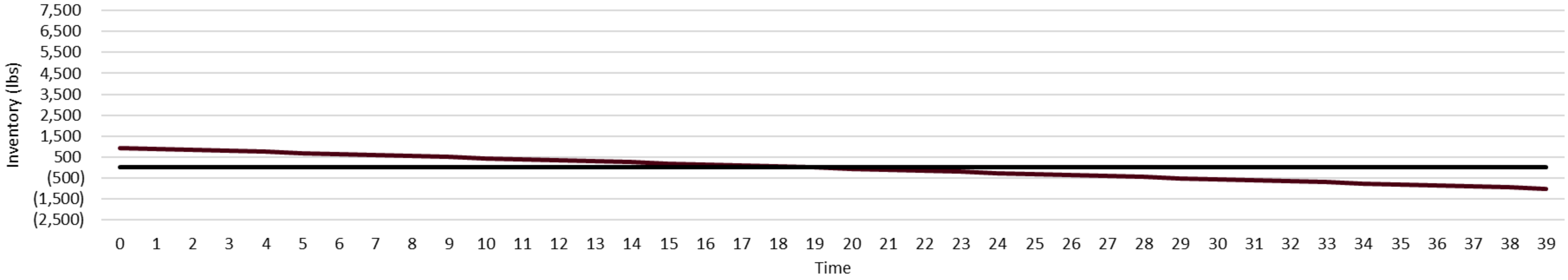
Case Study

→ Illustrative Examples

Scenario	Starting Inventory (lbs)	Forecast (lbs)	Running Time (min)	Optimal Solution	Cost per Production Job	MIP Gap	Production Jobs	Amount Produced (lbs)	Reorder Point (lbs)	Average Lead Time (hrs)
1	-	2,000	4.19	$ 1,816.32	$ 1,816.32	0.00%	1	2,434.58	450.00	9.00
2	-	10,000	8.19	$ 4,891.98	$ 2,445.99	4.33%	2	10,966.66	3,125.00	12.50
3	500	10,000	16.24	$ 4,682.65	$ 2,341.33	3.58%	2	10,966.66	3,250.00	13.00
4	1,000	10,000	20.95	$ 4,593.32	$ 2,296.66	2.93%	2	10,966.66	3,125.00	12.55
5	2,500	10,000	15.62	$ 3,635.33	$ 3,635.33	1.71%	1	937.31	2,750.00	11.00
6	500	2,000	3.81	$ 1,729.41	$ 1,729.41	0.00%	1	2,434.58	950.00	19.00
7	1,000	2,000	6.74	$ 1,240.00	-	3.88%	0	-	-	-
8	2,500	2,000	0.02	$ 1,180.00	-	0.00%	0	-	-	-

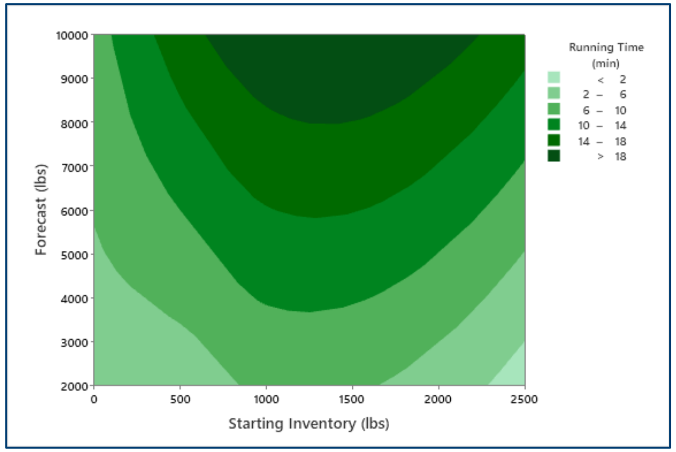
Scenario 5
Starting Inventory = 1,000 lbs | Forecast = 2,000 lbs

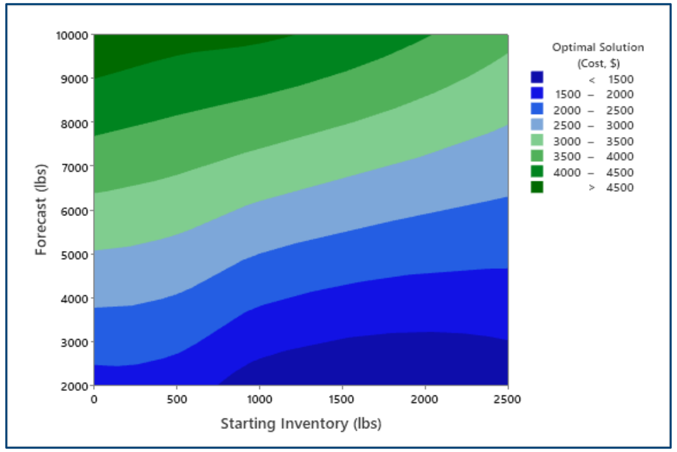
		Week	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		Day	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5
		Hour	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
Scenario	Job	T	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
7	0	Extrude																																								
		Dry																																								
		Coat																																								
		Package																																								

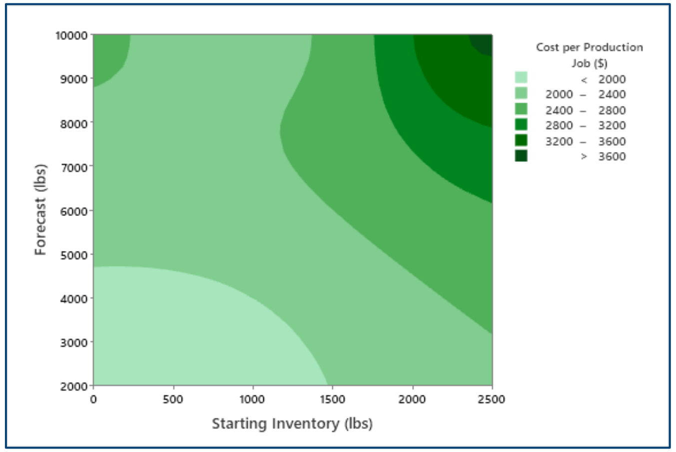


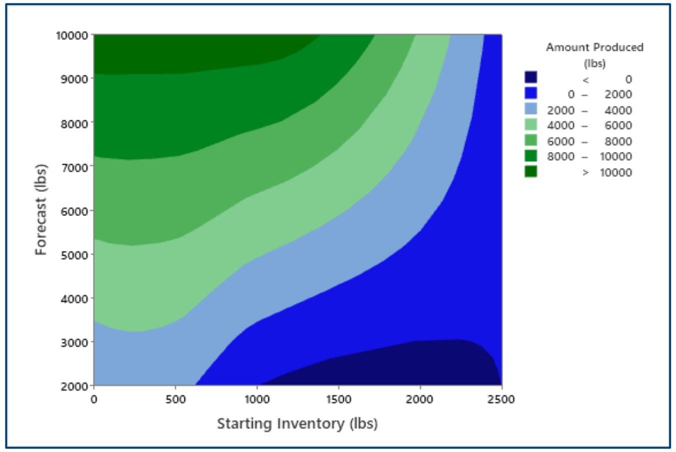
Case Study

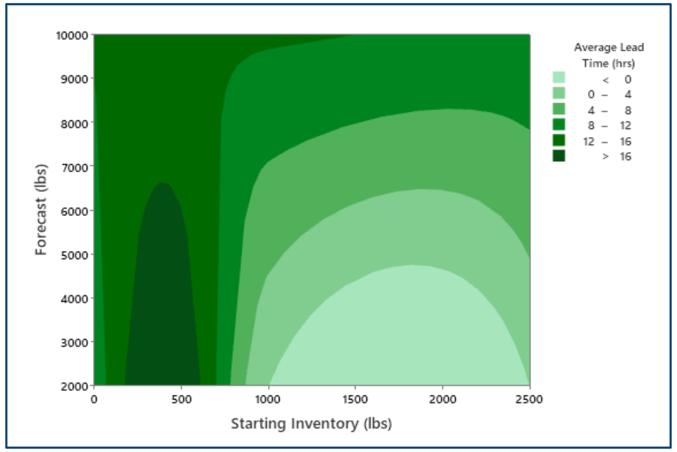
→ Numerical Analysis: Impact of Forecast and Starting Inventory

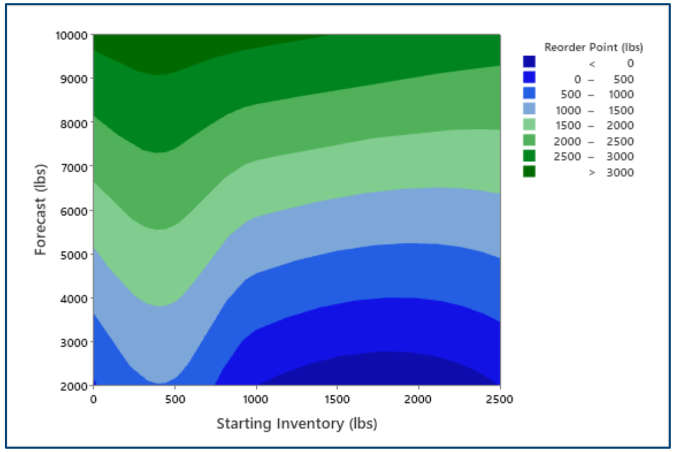












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Future Research

- Combine recipes to finished goods
- Include additional decision-making scenarios
  - Work schedules
  - Capacity
- Develop model for perishability
- Improve computational efficiency
  - Capability to include >1,000 SKUs and >400 recipes
  - Lengthen forecast horizon to 1-month
- Incorporate larger data set
  - Actual cost data
  - Actual inventory data
  - Actual forecast data
- Build GUI
  - User input
  - Visualizations

References

[1] United States Department of Agriculture, “Food Waste FAQs.” Accessed: Nov. 07, 2024. [Online]. Available: https://www.usda.gov/foodwaste/faqs

[2] Environmental Protection Agency, “2019 Wasted Food Report,” 2019.

[3] C. Cipolat-Gotet, A. Cecchinato, M. De Marchi, and G. Bittante, “Factors affecting variation of different measures of cheese yield and milk nutrient recovery from an individual model cheese-manufacturing process,” J Dairy Sci, vol. 96, no. 12, pp. 7952–7965, Dec. 2013, doi: 10.3168/jds.2012-6516.

[4] D. Somsen, A. Capelle, and J. Tramper, “Production yield analysis in the poultry processing industry,” J Food Eng, vol. 65, no. 4, pp. 479–487, Dec. 2004, doi: 10.1016/j.jfoodeng.2004.02.008.

[5] https://www.un.org/en/observances/end-food-waste-day

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Appendix

Proposed Model

→ Parameters

Availability

Demand

Inventory

Perishability

Costs

Production

Notation	Description
κ	Working weeks per year
λ	Working days per week
μ	Shifts per day
ν	Hours per shift
T	Time horizon

Proposed Model

→ Parameters

Availability

Demand

Inventory

Perishability

Costs

Production

Notation	Description
$\mathbb{d}_{jt}$	Demand of finished good j at time t
$\mathbb{D}_{j}$	Annualized demand of finished good j
$\bar{\mathbb{d}}_{j}$	Average demand of finished good j
σ_j	Standard deviation of demand of finished good j

Proposed Model
→ Parameters

Availability
Demand
Inventory
Perishability
Costs
Production

Notation	Description
Z	Z-score (based on service level, ψ_j)
ψ_j	Desired service level of finished good j
R_{jt}^{safe}	Safety inventory level of finished good j at time t

Proposed Model

→ Parameters

Availability
Demand
Inventory
Perishability
Costs
Production

Notation	Description
$\mathbb{S}_j$	Inventory shelf-life of finished good j
$\mathbb{s}_i$	Stockout cost

Proposed Model

→ Parameters

Availability
Demand
Inventory
Perishability
Costs
Production

Notation	Description	Value
h_j	Inventory holding cost of finished good j	\$0.02
h_j^S	Stockout cost of finished good j	\$0.10
$\mathbb{c}^{Setup}$	Generalized equipment setup cost per hour	\$29.53
$\mathbb{c}^{Labor}$	Generalized labor cost per hour	\$28.55
$\mathbb{c}_j^{Safety}$	Cost per unit of additional safety stock of finished good j	\$0.02
$\mathbb{c}_{Waste}^{Ext}$	Waste cost per unit of extrusion	\$0.02
$\mathbb{c}_{Waste}^{Dry}$	Waste cost per unit of drying	\$0.02
$\mathbb{c}_{Waste}^{Coat}$	Waste cost per unit of coating	\$0.02
$\mathbb{c}_{Waste}^{Pack}$	Waste cost per unit of packaging	\$0.02
$\mathbb{c}_j^{Order}$	Order cost per unit of finished good j	\$0.02
$\mathbb{c}_j^{FG}$	Cost per unit of finished good j	\$0.02
$\mathbb{c}_l^{Ext}$	Production cost per hour of extruder machine l	\$1.00
$\mathbb{c}_d^{Dry}$	Production cost per hour of drying machine d	\$1.00
$\mathbb{c}_c^{Coat}$	Production cost per hour of coating machine c	\$1.00
$\mathbb{c}_m^{Pack}$	Production cost per hour of packaging machine m	\$1.00

Proposed Model

→ Parameters

Availability
Demand
Inventory
Perishability
Costs
Production

Notation	Description	Value
P_{jt}	Amount produced of finished good j during time t	
α_{jl}	Production rate for finished good j on extruder machine l	
β_{jd}	Production rate for finished good j on drying machine d	60,000 tons per week
γ_{jc}	Production rate for finished good j on coating machine c	60,000 tons per week
δ_{jm}	Production rate for finished good j on packing machine m	
ξ	Production yield on extruder machine l	100%
ν	Production yield on drying machine d	100%
φ	Production yield on coating machine c	100%
χ	Production yield on packing machine m	100%

Proposed Model

→ Parameters

Availability
Demand
Inventory
Perishability
Costs
Production

Notation	Description	Value
η_{jl}^{-}	Input (batch size) of finished good j on extruder machine l	Calc
η_{jl}^{+}	Output of finished good j on extruder machine l ; $\eta_{jl}^{+} = \eta_{jl}^{-} * \xi$	Calc
ε_{jl}^{-}	Input of finished good j on drying machine after extruder machine l ; $\varepsilon_{jl}^{-} = \eta_{jl}^{+}$	Calc
ζ_{jl}^{-}	Input of finished good j on coating machine after extruder machine l ; $\zeta_{jl}^{+} = \zeta_{jl}^{-} * \varphi$	Calc
ϑ_{jl}^{-}	Input of finished good j on packing machine after extruder machine l ; $\vartheta_{jl}^{-} = \zeta_{jl}^{+}$	Calc
ϑ_{jl}^{+}	Output of finished good j on packing machine after extruder machine l ; $\vartheta_{jl}^{+} = \vartheta_{jl}^{-} * \chi$	Calc
p_{ln}	Setup time of extruder machine l for changeover n	10 minutes
q_{dn}	Setup time of drying machine d for changeover n	10 minutes
r_{cn}	Setup time of coating machine c for changeover n	10 minutes
s_{mn}	Setup time of packing machine m for changeover n	10 minutes

Proposed Model

→ Parameters

Availability
Demand
Inventory
Perishability
Costs
Production

Notation	Description
τ_{jl}	Run time for finished good j on extruder machine l; $\tau_{jl} = \mathscr{p}_{ln} + \frac{\eta_{jl}^{-}}{\alpha_{jl}}$
ϕ_{jld}	Run time for finished good j on drying machine d after extruder machine l; $\phi_{jld} = \mathscr{q}_{dn} + \frac{\varepsilon_{jl}^{-}}{\beta_{jd}}$
ω_{jlc}	Run time for finished good j on coating machine c after extruder machine l; $\omega_{jlc} = \mathscr{r}_{cn} + \frac{\zeta_{jl}^{-}}{\gamma_{jc}}$
ρ_{jlm}	Run time for finished good j on packing machine m after extruder machine l; $\rho_{jlm} = \mathscr{s}_{mn} + \frac{\vartheta_{jl}^{+}}{\delta_{jm}}$
n_j	Total number of jobs required to satisfy the demand of finished good j for the entire period; $\frac{\mathbb{D}_j}{\min(\vartheta_{jl}^{+})}$

Proposed Model

→ Decision Variables

Start Time

Start and Finish Times

Machine Assignment

Inventory

Notation	Description
x_{klt}	Takes the value of 1 if job k starts on extruder machine l at time t; 0 otherwise
y_{kldt}	Takes the value of 1 if job k starts on drying machine d at time t after extruder machine l; 0 otherwise
z_{klct}	Takes the value of 1 if job k starts on coating machine c at time t after extruder machine l; 0 otherwise
w_{klmt}	Takes the value of 1 if job k starts on packing machine m at time t after extruder machine l; 0 otherwise

Proposed Model

→ Decision Variables

Start Time

Start and Finish Times

Machine Assignment

Inventory

Notation	Description
τ_k^-	Start time of job k on extruder machine
τ_k^+	Finish time of job k on extruder machine
ϕ_k^-	Start time of job k on drying machine
ϕ_k^+	Finish time of job k on drying machine
ω_k^-	Start time of job k on coating machine
ω_k^+	Finish time of job k on coating machine
ρ_k^-	Start time of job k on packing machine
ρ_k^+	Finish time of job k on packing machine
φ_{kt}^-	Takes the value of 1 if job k starts at time t; 0 otherwise
φ_{kt}^+	Takes the value of 1 if job k finishes at time t; 0 otherwise
γ_{kt}	Takes the value of 1 if job k can be finished within time T; 0 otherwise
δ_{jt}	Takes the value of 1 if product j need to starts a job at time t; 0 otherwise
e_{kt}	Takes the value of 1 if job k is still in the work in process at time t; 0 otherwise
ϵ_k	Start time of job k

Proposed Model
→ Decision Variables

Start Time

Start and Finish Times

Machine Assignment

Inventory

Notation	Description
u_{klt}	Takes the value of 1 if job k is assigned to extruder machine l at time t; 0 otherwise
o_{kldt}	Takes the value of 1 if job k is assigned to drying machine d at time t after extruder machine l; 0 otherwise
g_{klct}	Takes the value of 1 if job k is assigned to coating machine c at time t after extruder machine l; 0 otherwise
p_{klmt}	Takes the value of 1 if job k is assigned to packing machine m at time t after extruder machine l; 0 otherwise

Proposed Model

→ Decision Variables

Start Time

Start and Finish Times

Machine Assignment

Inventory

Notation	Description
$\mathbb{I}_{jt}$	Inventory levels of finished good j at time t
$\mathbb{I}_{jt}^{S}$	Stockout levels of finished good j at time t
$\bar{\mathcal{L}}_j$	Average lead time of finished good j
$\mathbb{d}_j$	Demand of finished good j during replenishment lead time
$\sigma_j^{\bar{\mathcal{L}}_j}$	Standard deviation of demand of finished good j during replenishment lead time
s_j	Safety stock of finished good j
R_j	Reorder point of finished good j

Proposed Model

→ Constraints

Task Assignment

Sequencing, Precedence, and Runtime

Job Initiation

Inventory

Perishability

Notation	Description
$\sum_k u_{klt} \leq 1, \forall l \in L, \forall t \in T$	Extruder machine l can only be assigned one job k in each period t
$\sum_k \sum_l o_{kldt} \leq 1, \forall d \in D, \forall t \in T$	Drying machine d can only be assigned one job k in each period t
$\sum_k \sum_l g_{klct} \leq 1, \forall c \in C, \forall t \in T$	Coating machine c can only be assigned one job k in each period t
$\sum_k \sum_l p_{klmt} \leq 1, \forall m \in M, \forall t \in T$	Packing machine m can only be assigned one job k in each period t

Proposed Model

→ Constraints

Task Assignment

Sequencing, Precedence, and Runtime

Job Initiation

Inventory

Perishability

Notation	Description
$\epsilon_k \geq 0 \,, \forall k \in K$	The start time of job k on the extruder machine l must be before or at to the time job is initiated
$\phi_k^- \geq \epsilon_k \,, \forall k \in K$	The start time of job k on extruder machine l must be greater or equal to the time job is initiated
$\phi_k^- \geq \tau_k^+ + \sum_{t \in T} \varphi_{kt}^- \,, \forall k \in K$	The start time of job k on drying machine d must be greater than the finish time of job k on extruder machine l
$\omega_k^- \geq \phi_k^+ + \sum_{t \in T} \varphi_{kt}^- \,, \forall k \in K$	The start time of job k on coating machine c must be greater than the finish time of job k on drying machine d
$\rho_k^- \geq \omega_k^+ + \sum_{t \in T} \varphi_{kt}^- \,, \forall k \in K$	The start time of job k on packing machine m must be greater than the finish time of job k on coating machine c
$\rho_k^T \leq T \,, \forall k \in K$	The finish time of job k on packing machine m must be within T

Proposed Model
→ Constraints

Task Assignment
Sequencing, Precedence, and Runtime
Job Initiation
Inventory
Perishability

Machine Assignment Decision

$$\sum_{l \epsilon L}\left(\sum_{t=0}^{T-\tau_{jl}} x_{klt}\right) = \sum_{t \in T} \varphi_{\bar{kt}} \;, \forall k \in K$$

$$\sum_{d \epsilon D}\left(\sum_{t=0}^{T-\phi_{jld}} y_{kldt}\right) = \sum_{t=0}^{T-\tau_{jl}} x_{klt} \;, \forall k \in K, \forall l \in L$$

$$\sum_{c \epsilon C}\left(\sum_{t=0}^{T-\omega_{jlc}} z_{klct}\right) = \sum_{t=0}^{T-\tau_{jl}} x_{klt} \;, \forall k \in K, \forall l \in L$$

$$\sum_{m \epsilon M}\left(\sum_{t=0}^{T-\rho_{jlm}} w_{klmt}\right) = \sum_{t=0}^{T-\tau_{jl}} x_{klt} \;, \forall k \in K, \forall l \in L$$

Proposed Model
→ Constraints

Task Assignment

Sequencing, Precedence, and Runtime

Job Initiation

Inventory

Perishability

Job Starting and Processing Decisions for Each Stage

$$\sum_{t\in T} u_{klt} = \tau_{jl} * \sum_{t\in T} x_{klt} \ , \forall k \in K, \forall l \in L$$

$$\sum_{t\in T} o_{kldt} = \phi_{jld} * \sum_{t\in T} y_{kldt} \ , \forall k \in K, \forall l \in L, \forall d \in D, \forall j \in J$$

$$\sum_{t\in T} g_{klct} = \omega_{jlc} * \sum_{t\in T} z_{klct} \ , \forall k \in K, \forall l \in L, \forall c \in C, \forall j \in J$$

$$\sum_{t\in T} p_{klmt} = \rho_{jlm} * \sum_{t\in T} w_{klmt} \ , \forall k \in K, \forall l \in L, \forall m \in M, \forall j \in J$$

$$\sum_{t'=t}^{t'+\tau_{jl}-1} u_{klt'} \geq \tau_{jl}\, x_{klt} \ , \forall k \in K, \forall l \in L, \forall t \in [0, T-\tau_{jl}]$$

$$\sum_{t'=t}^{t'+\phi_{jld}-1} o_{kldt'} \geq \phi_{jld}\, y_{kldt} \ , \forall k \in K, \forall l \in L, \forall d \in D, \forall t \in [0, T-\phi_{jld}]$$

$$\sum_{t'=t}^{t'+\omega_{jlc}-1} g_{klct'} \geq \omega_{jlc}\, z_{klct} \ , \forall k \in K, \forall l \in L, \forall c \in C, \forall t \in [0, T-\omega_{jlc}]$$

$$\sum_{t'=t}^{t'+\rho_{jlm}-1} p_{klmt'} \geq \rho_{jlm}\, w_{klmt} \ , \forall k \in K, \forall l \in L, \forall m \in M, \forall t \in [0, T-\rho_{jlm}]$$

Proposed Model

→ Constraints

Task Assignment

Sequencing, Precedence, and Runtime

Job Initiation

Inventory

Perishability

Determine Start and End Times of Each Job on Each Machine

$$\tau_k^- = T\left(1 - \sum_{t \in T} \varphi_{kt}^-\right) + \sum_{t \in T}\left(t * \sum_{l \epsilon L} x_{klt}\right), \forall k \in K$$

$$\phi_k^- = T\left(1 - \sum_{t \in T} \varphi_{kt}^-\right) + \sum_{t \in T}\left(t * \sum_{l \epsilon L} \sum_{d \epsilon D} y_{kldt}\right), \forall k \in K$$

$$\omega_k^- = T\left(1 - \sum_{t \in T} \varphi_{kt}^-\right) + \sum_{t \in T}\left(t * \sum_{l \epsilon L} \sum_{c \epsilon C} z_{klct}\right), \forall k \in K$$

$$\rho_k^- = T\left(1 - \sum_{t \in T} \varphi_{kt}^-\right) + \sum_{t \in T}\left(t * \sum_{l \epsilon L} \sum_{m \epsilon M} w_{klmt}\right), \forall k \in K$$

$$\tau_k^+ = \tau_k^- - \sum_{t \in T} \varphi_{kt}^- + \sum_{l \epsilon L} \sum_{t \epsilon T} u_{klt}, \forall k \in K$$

$$\phi_k^+ = \phi_k^- - \sum_{t \in T} \varphi_{kt}^- + \sum_{l \epsilon L} \sum_{d \epsilon D} \sum_{t \epsilon T} o_{kldt}, \forall k \in K$$

$$\omega_k^+ = \omega_k^- - \sum_{t \in T} \varphi_{kt}^- + \sum_{l \epsilon L} \sum_{c \epsilon C} \sum_{t \epsilon T} g_{klct}, \forall k \in K$$

$$\rho_k^+ = \rho_k^- - \sum_{t \in T} \varphi_{kt}^- + \sum_{l \epsilon L} \sum_{m \epsilon M} \sum_{t \epsilon T} p_{klmt}, \forall k \in K$$

Proposed Model

→ Constraints

Task Assignment
Sequencing, Precedence, and Runtime
Job Initiation
Inventory
Perishability

Determine Job Initiation and Processing Decisions

$$\sum_{t\in T} \varphi_{kt}^{-} \leq 1, \forall k \in K$$

$$\sum_{k\in K^j} \varphi_{kt}^{-} \leq \delta_{jt}, \forall j \in J, \forall t \in T$$

$$\sum_{t\in T} \varphi_{kt}^{-} \leq \gamma_k, \forall k \in K$$

$$\sum_{t'=0}^{t} \varphi_{k+1,t}^{-} \geq \gamma_{k+1} + \frac{2}{T}\left(\sum_{t'=0}^{t} \delta_{jt'} - \sum_{k'=k^{j,0}}^{k} \sum_{t'=0}^{t} \varphi_{k',t'}^{-}\right) - 1, \forall k' \in K^j, \forall t \in T$$

$$\sum_{t'=0}^{t} \varphi_{k+1,t}^{-} \leq \sum_{t'=0}^{t} \varphi_{kt}^{-}, \forall k \in K^j, \forall t \in T$$

$$\sum_{t\in T} e_{kt} = \rho_k^{+} - \epsilon_k - \sum_{t\in T} \varphi_{kt}^{-}, \forall k \in K$$

$$\epsilon_k = \sum_{t\in T} t * \varphi_{kt}^{-} + T(1 - \sum_{t\in T} \varphi_{kt}^{-}), \forall k \in K$$

Proposed Model
→ Constraints

Task Assignment
Sequencing, Precedence, and Runtime
Job Initiation
Inventory
Perishability

Determine Production, Inventory, and Stockout Levels by Product

$$\sum_{t\in T} t * \varphi^{+}_{kt} = \rho^{+}_{k} \sum_{t\in T} \varphi^{-}_{kt} \text{ , } \forall k \in K$$

$$\sum_{t\in T} \varphi^{+}_{kt} = \sum_{t\in T} \varphi^{-}_{kt} \text{ , } \forall k \in K$$

$$P_{jt} = \sum_{k\in K_j} \left[\varphi^{+}_{k,t} * \sum_{l\in L} \left(\vartheta^{+}_{jl} * \sum_{t'\in T} x_{klt'} \right) \right] \text{ , } \forall j \in J, \forall t \in T$$

$$\mathbb{I}_{jt} - \mathbb{I}^{S}_{jt} = \mathbb{I}_{j,t-1} - \mathbb{I}^{S}_{j,t-1} - \mathbb{d}_{jt} + P_{jt} \text{ , } \forall j \in J, \forall t \in T$$

$$\mathbb{I}_{jt}, \mathbb{I}^{S}_{jt} \geq 0 \text{ , } \forall j \in J, \forall t \in T$$

$$R_j = \mathbb{d}_j + Z\sigma_j^{\bar{\mathcal{L}}_j}$$

$$\mathbb{d}_j = \bar{\mathbb{d}}_j \bar{\mathcal{L}}_j$$

$$\sigma_j^{\bar{\mathcal{L}}_j} = \sqrt{\sigma_j^2 \bar{\mathcal{L}}_j}$$

$$\bar{\mathcal{L}}_j = \frac{1}{\max(K^j, 1)} \sum_{k}^{K^j} (\rho^{+}_{k} - \epsilon_k)$$

Proposed Model

→ Constraints

Task Assignment

Sequencing, Precedence, and Runtime

Job Initiation

Inventory

Perishability

Notation	Description
$\mathbb{s}_j \geq \phi_k^+ - \tau_k^-$	The lead time from the start of extrusion to the end of drying must be less than or equal to the recipe's perishable life